Discussion Section 8

- Baum-Welch

- NP-completeness proofs (or how to say “actually, this probably can't be done efficiently”)

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$$\max_{p} P_{\theta}(p, S)$$

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- Viterbi likelihood:

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- Baum-Welch likelihood:

$$\sum_{p} P_{\theta}(p, S)$$

Baum-Welch

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2) Use **forward-backward** to get fractional counts for each edge type

    (total probability of paths passing through edge)/
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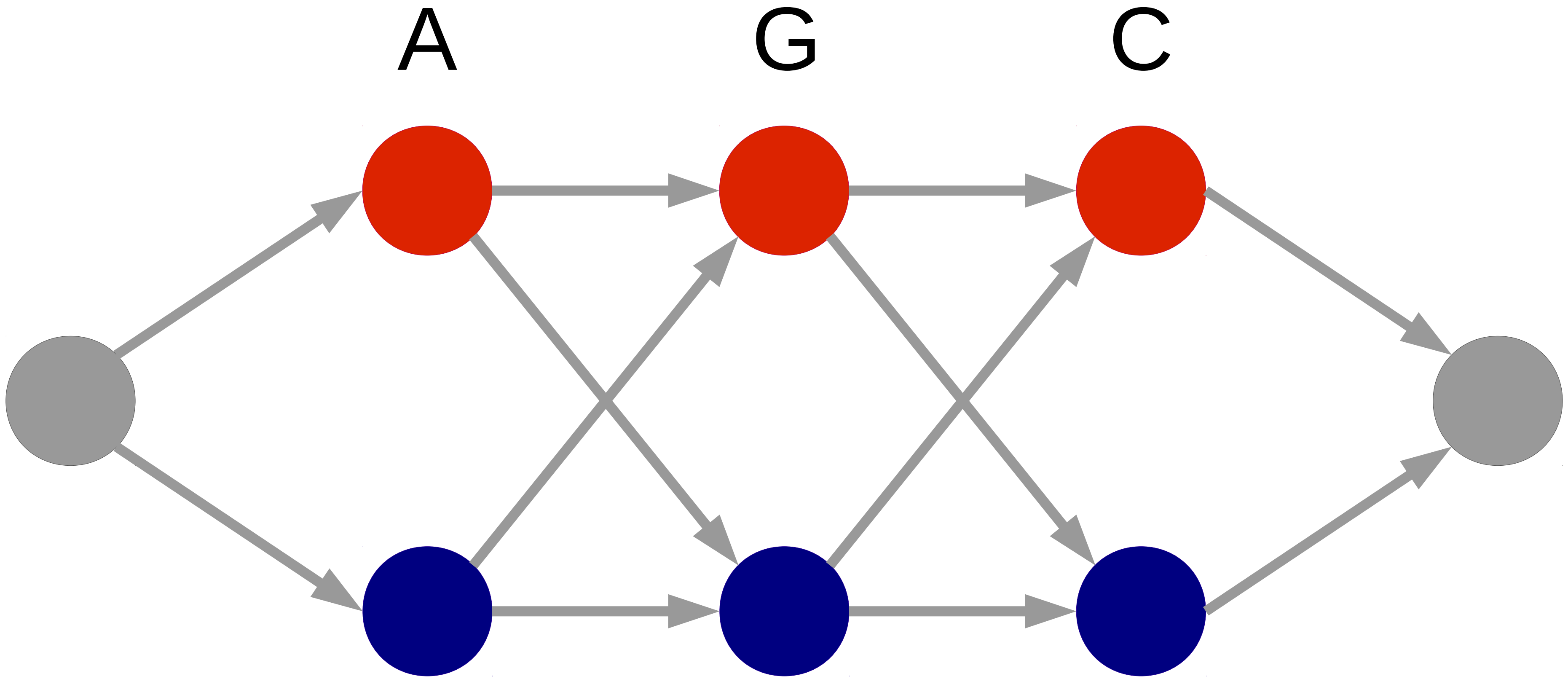
3) Re-estimate transition and emission probabilities by calculating the expected number of each edge type

Forward-backward algorithm



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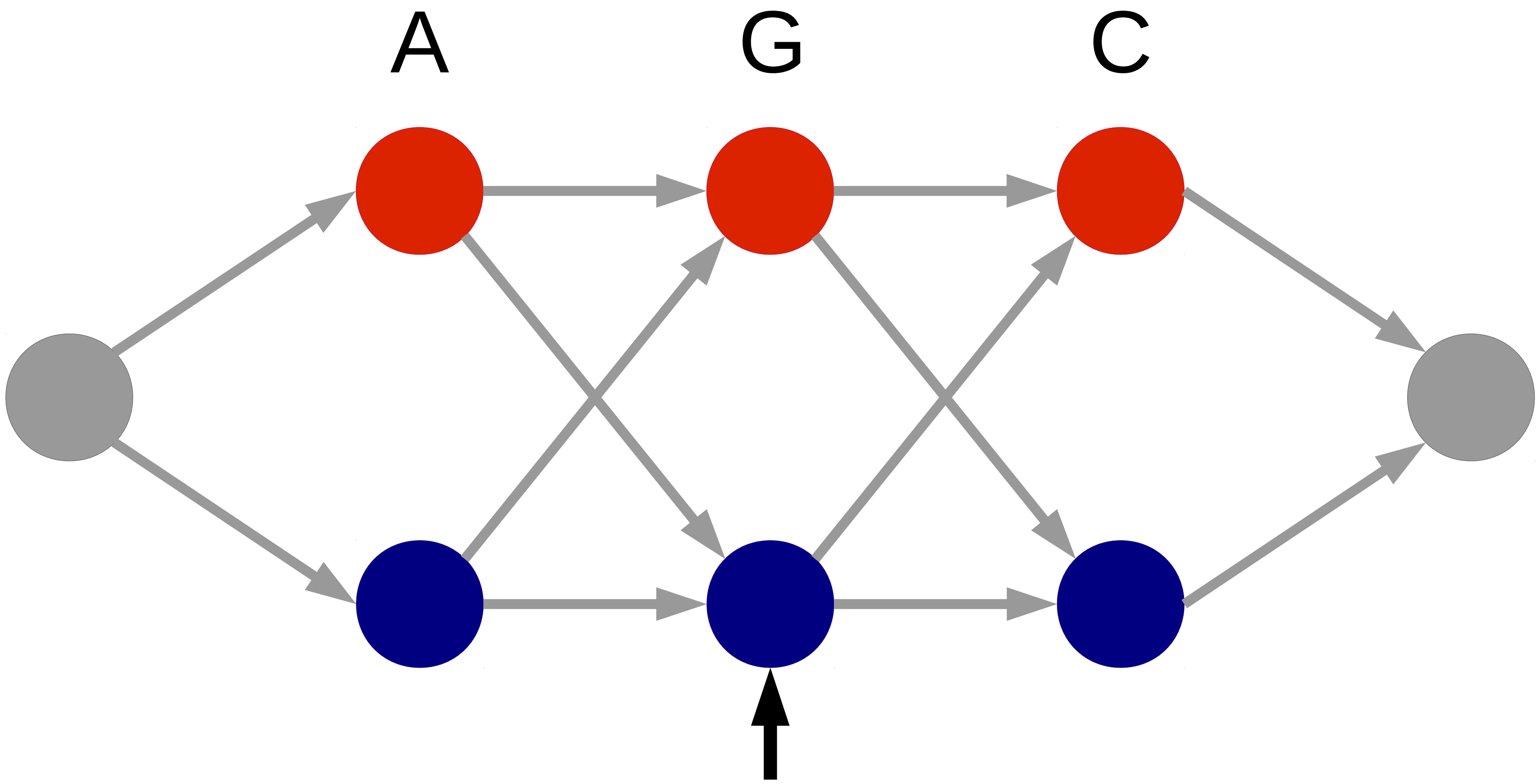

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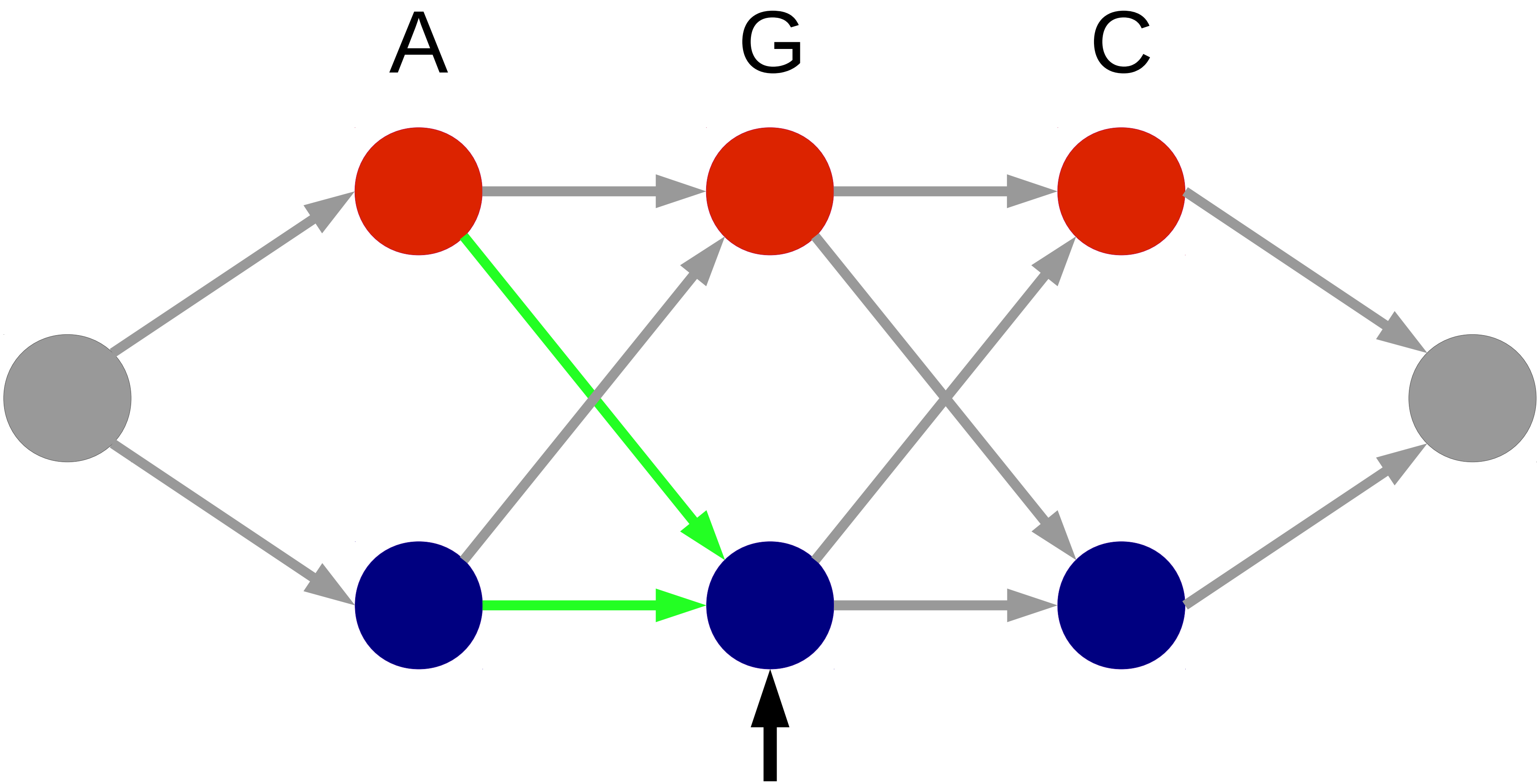

For each node:

- Forward: Store the sum of probabilities of paths ending at position *i* state *k*

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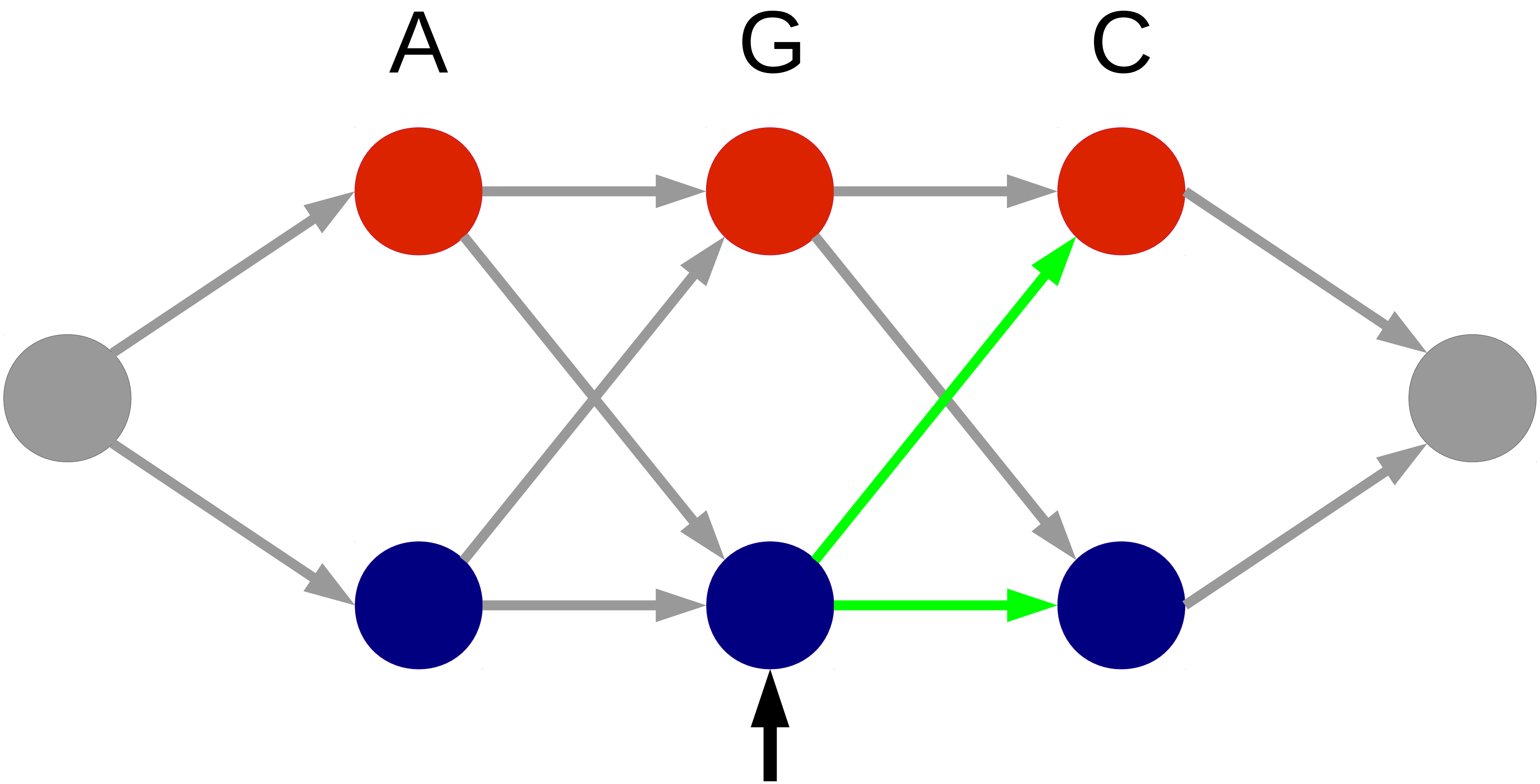

For each node:

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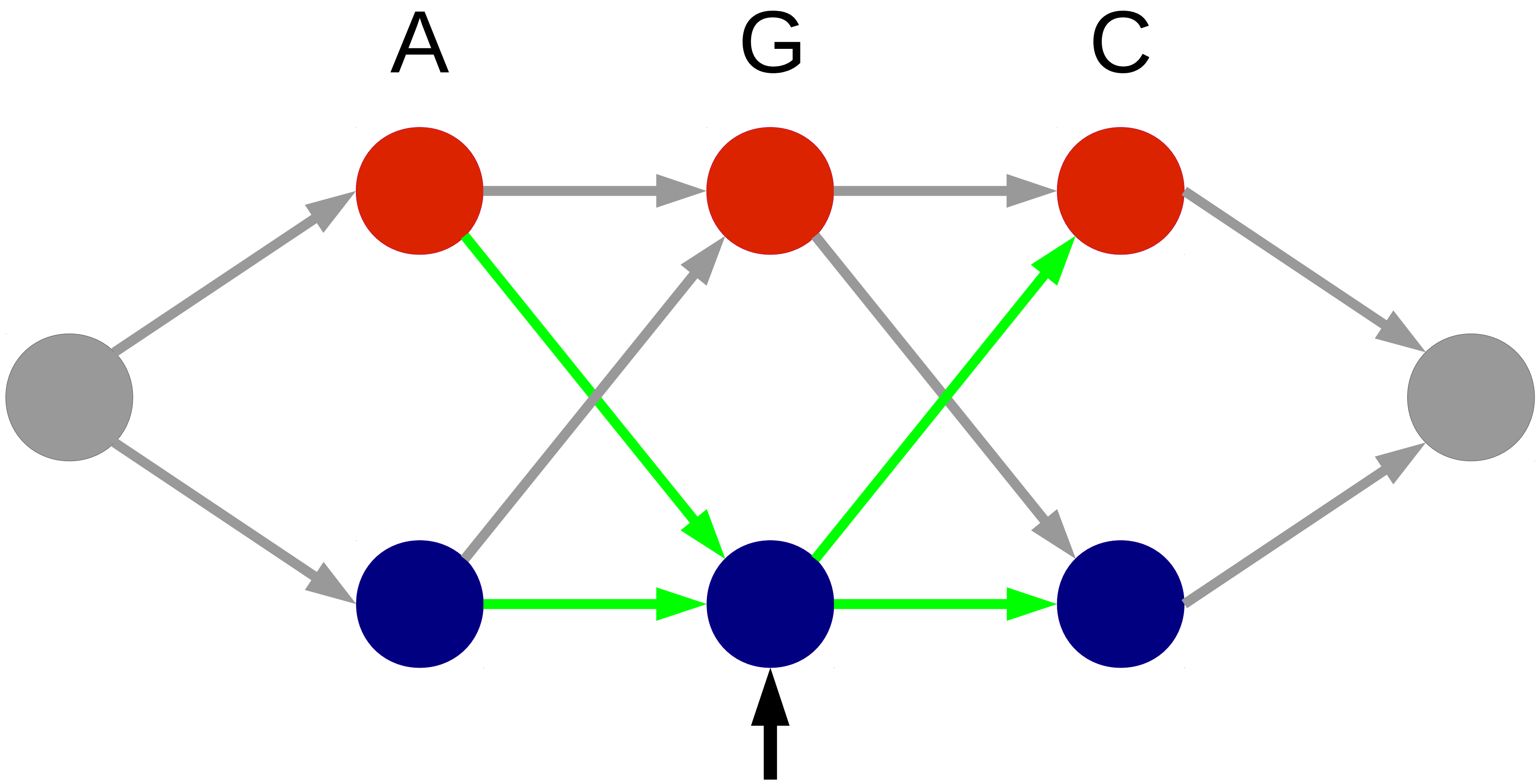

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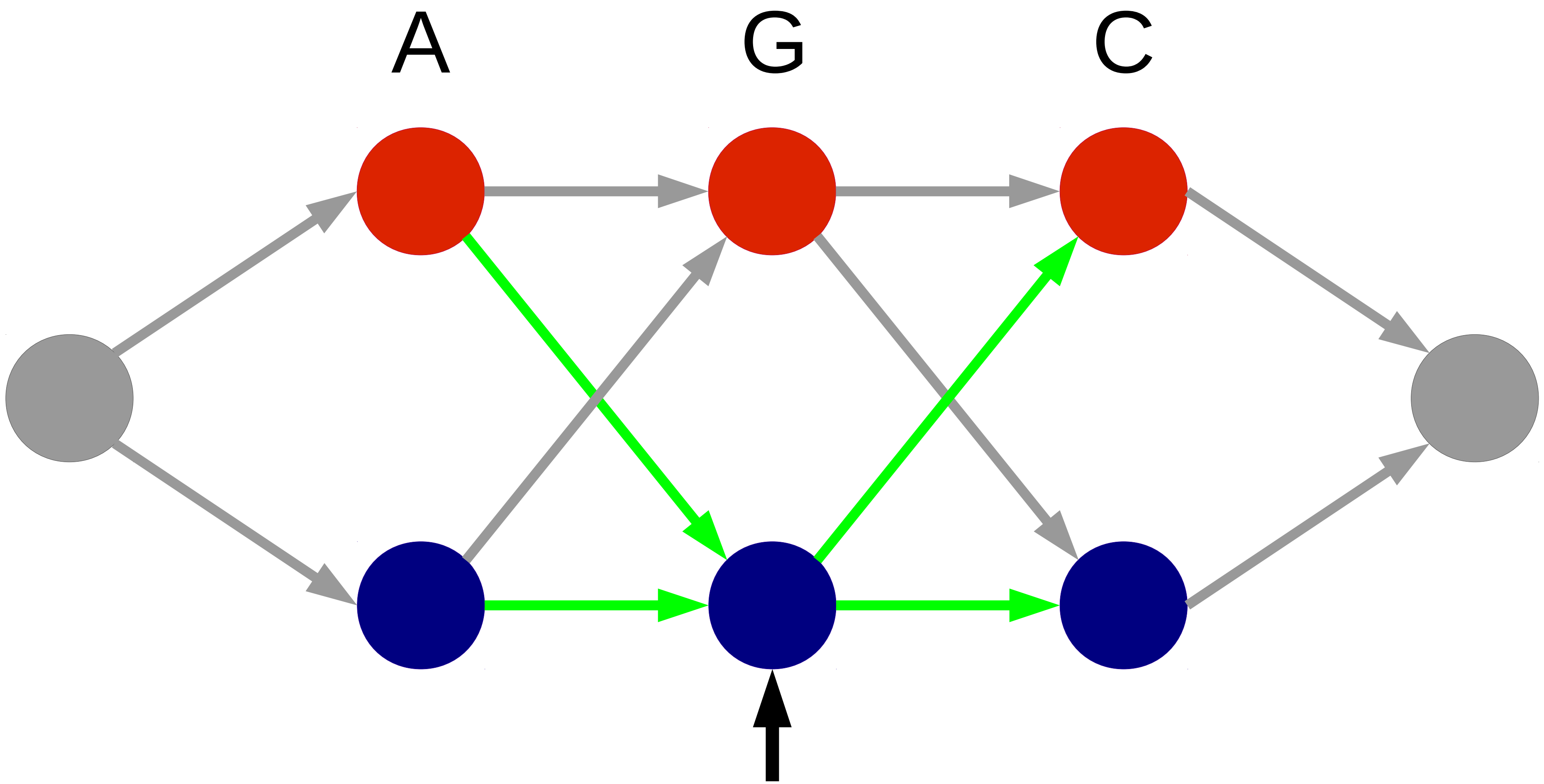
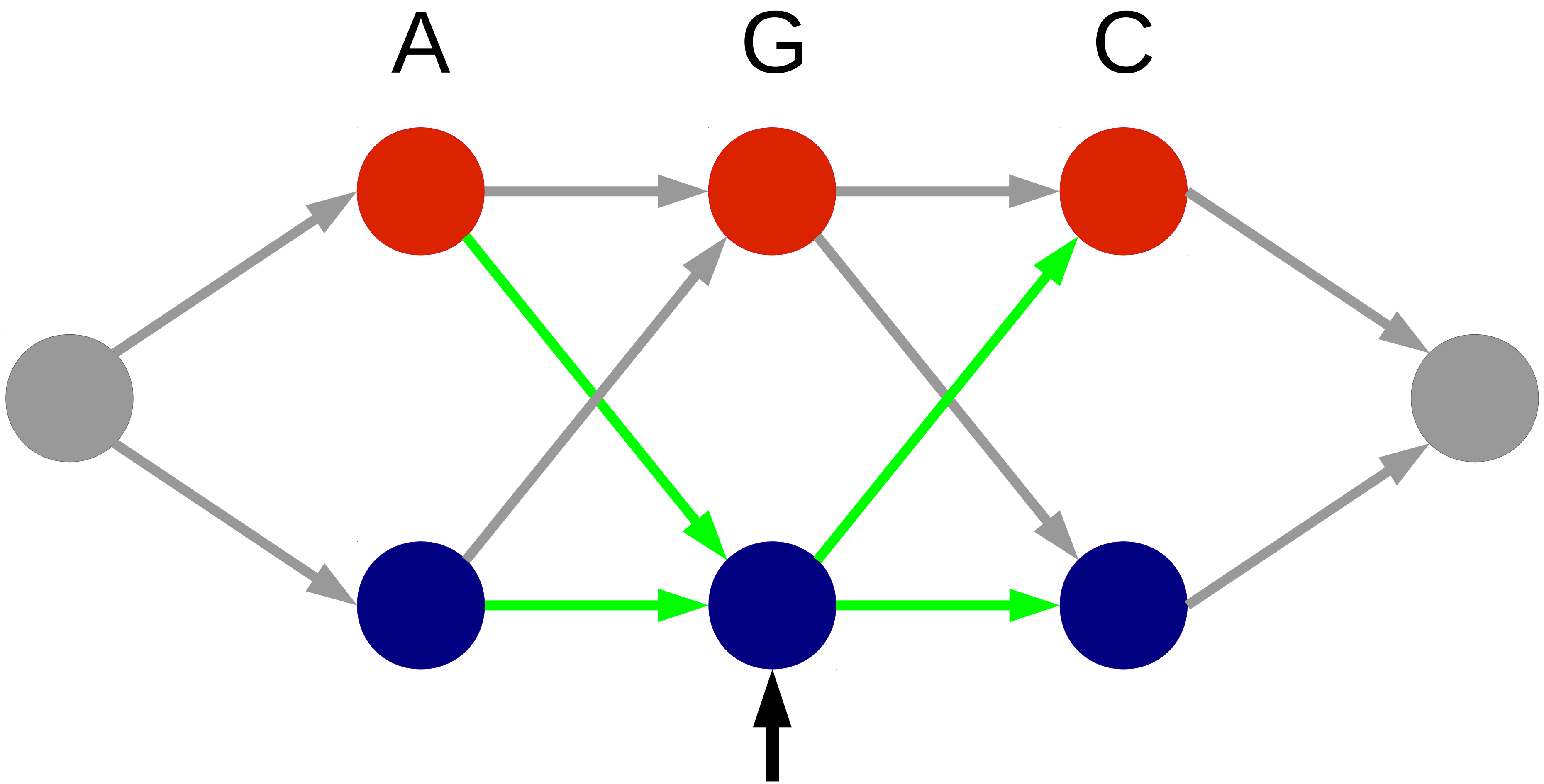


Total probability of paths passing through position *i* state *k*:

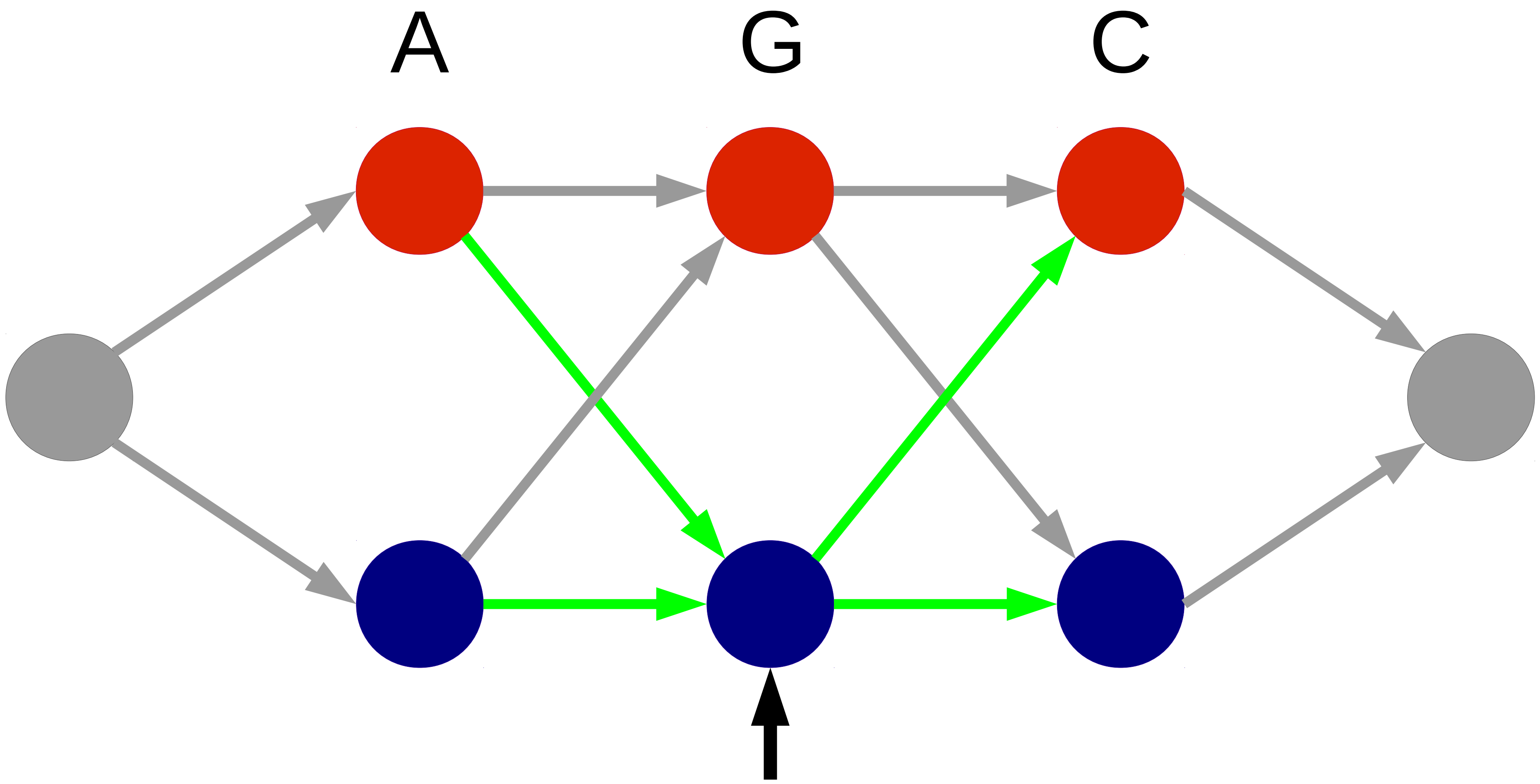
Forward-backward algorithm



Total probability of paths passing through position *i* state *k*:

- forward(*i*, *k*) x emission(S_i, *k*) x backward(*i*, *k*)

Forward-backward algorithm



Total probability of paths passing through position *i* state *k*:

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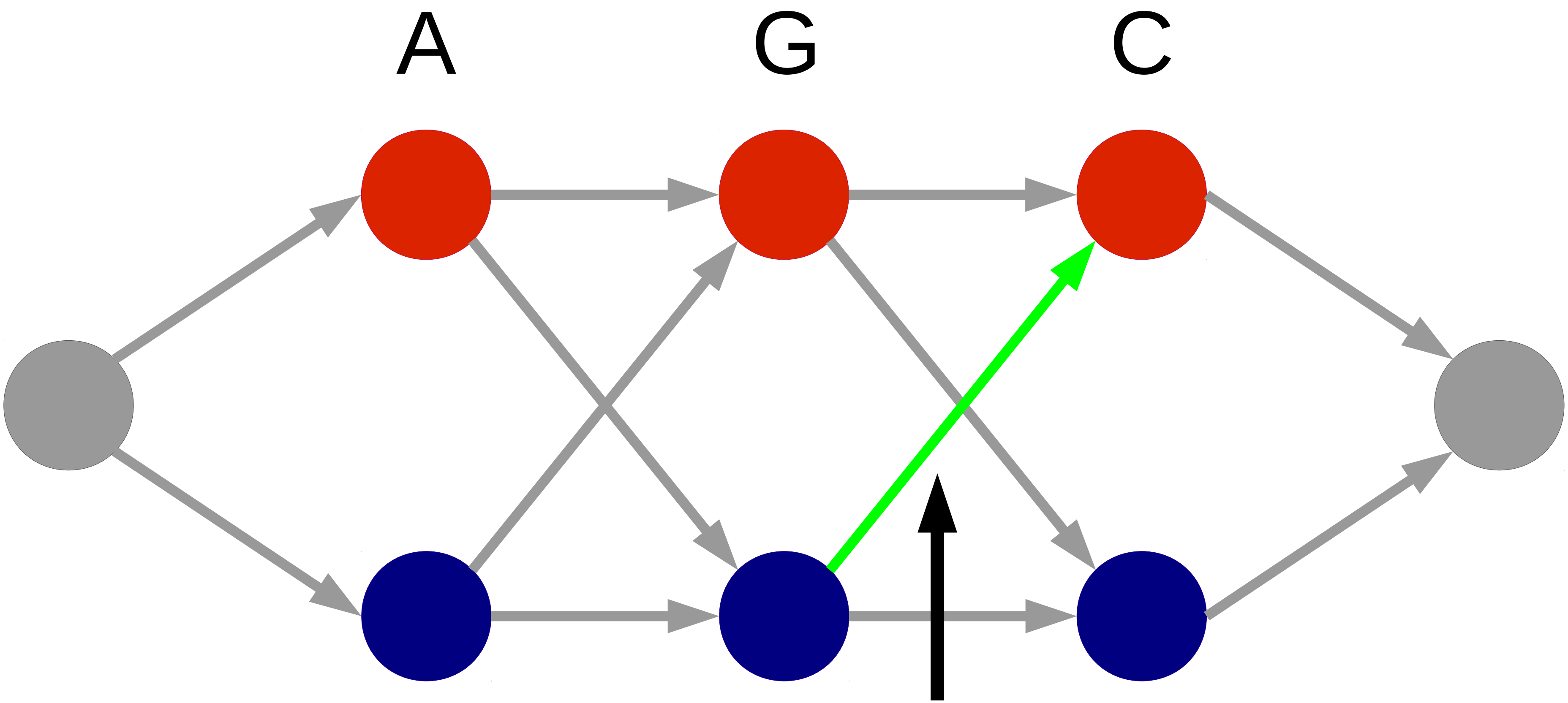
- In this example, add this weighted count to the numerator for the blue state emitting 'G' and the denominator for all blue state emission probabilities

Forward-backward algorithm



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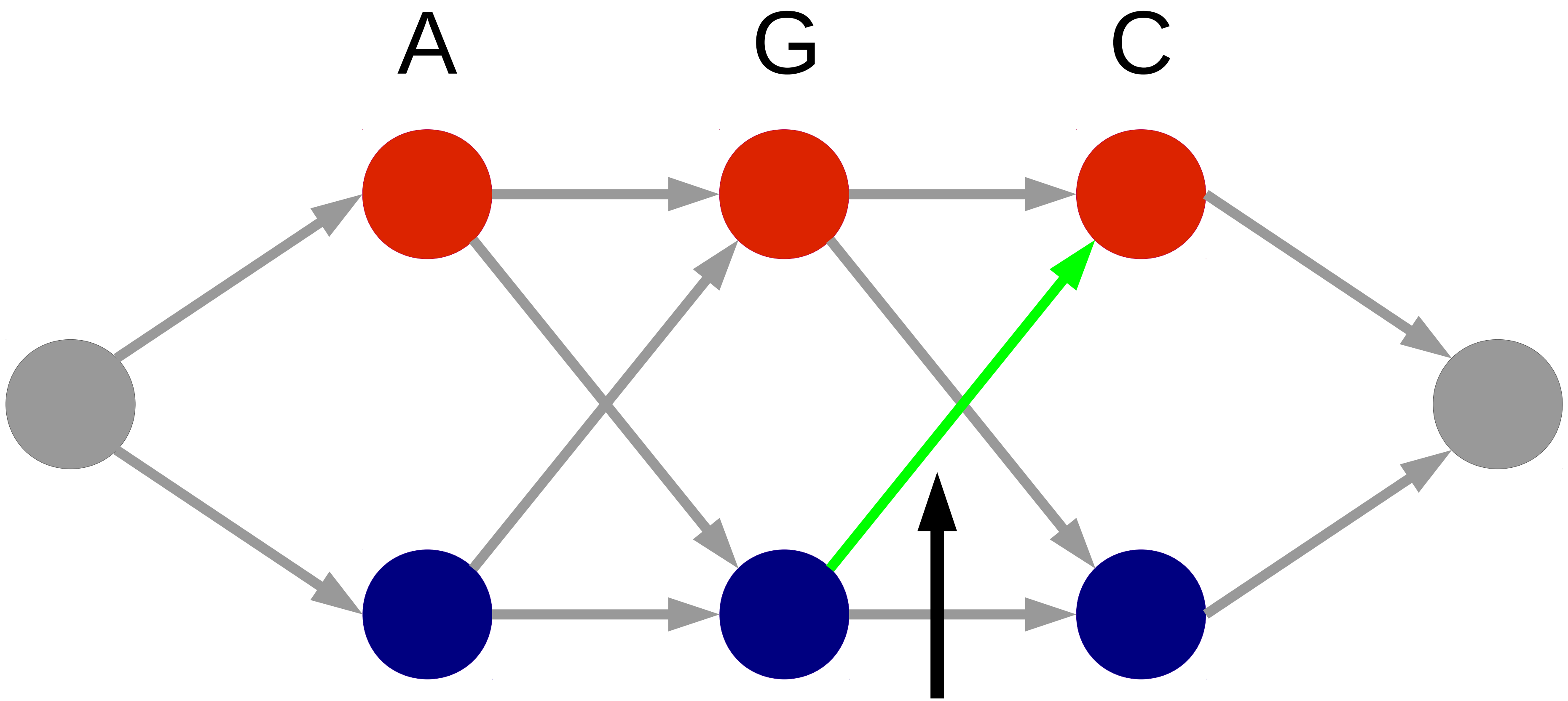

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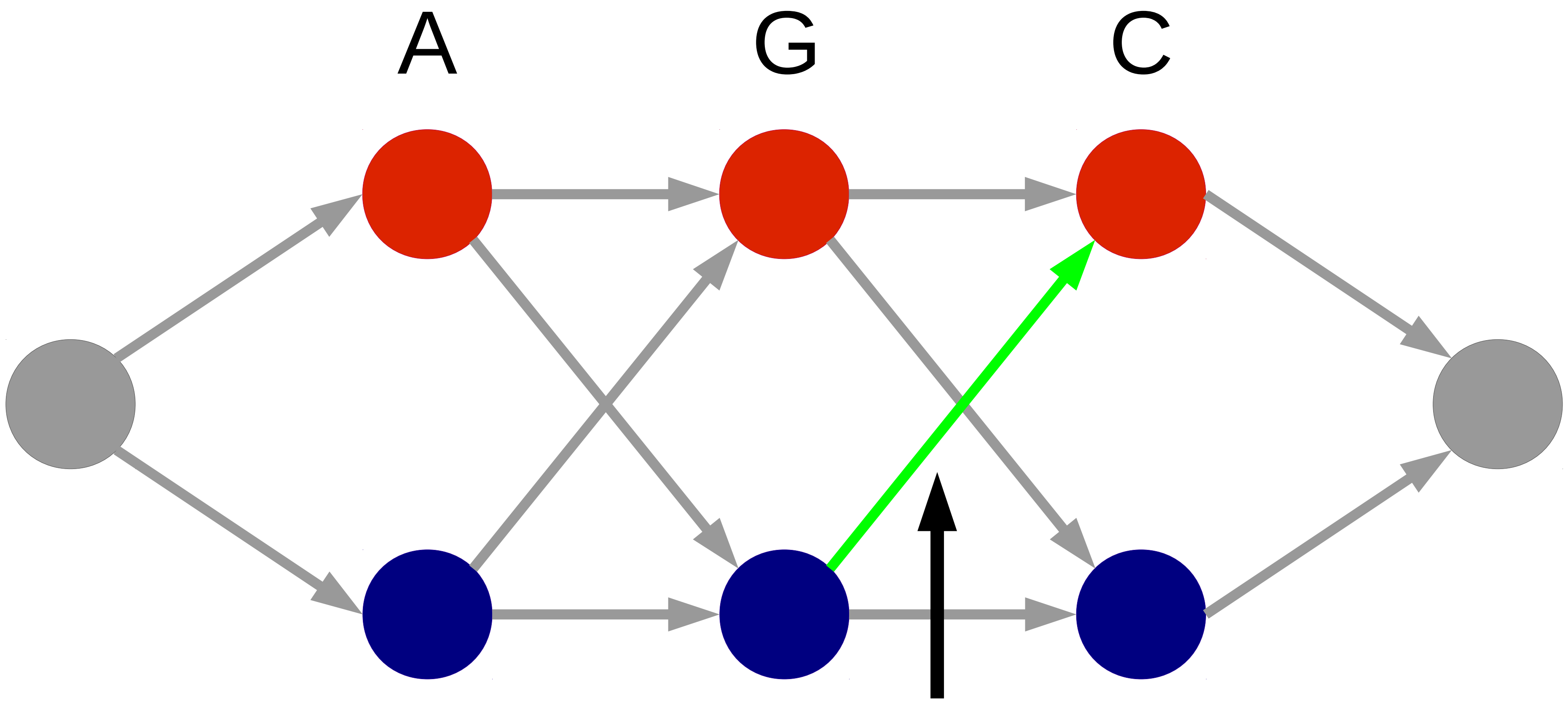

Total probability of paths passing from position *i*-1 state *k*' to position *i* state *k*:

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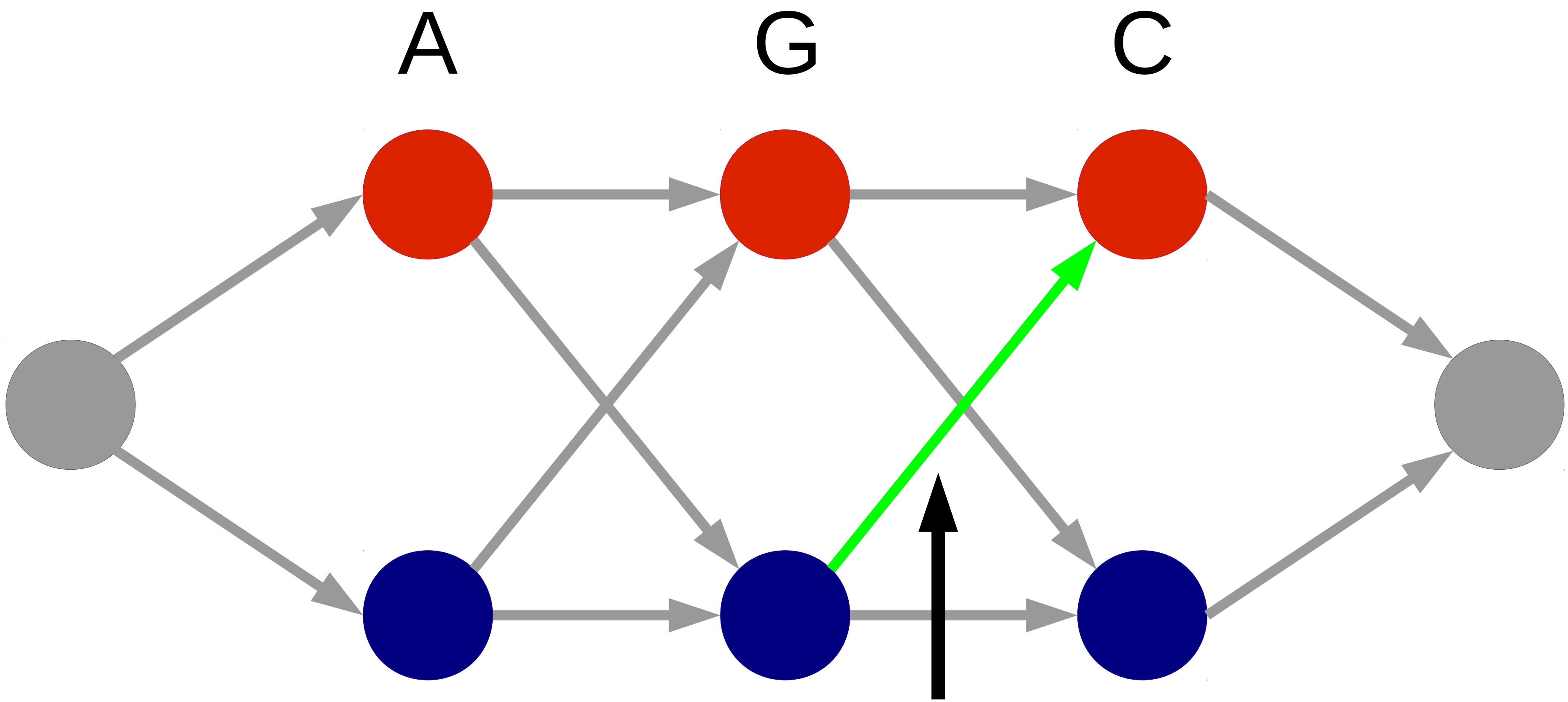

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An alternative way to think about updating

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  - a_{kl}: The transition probability from state *k* to state *l*

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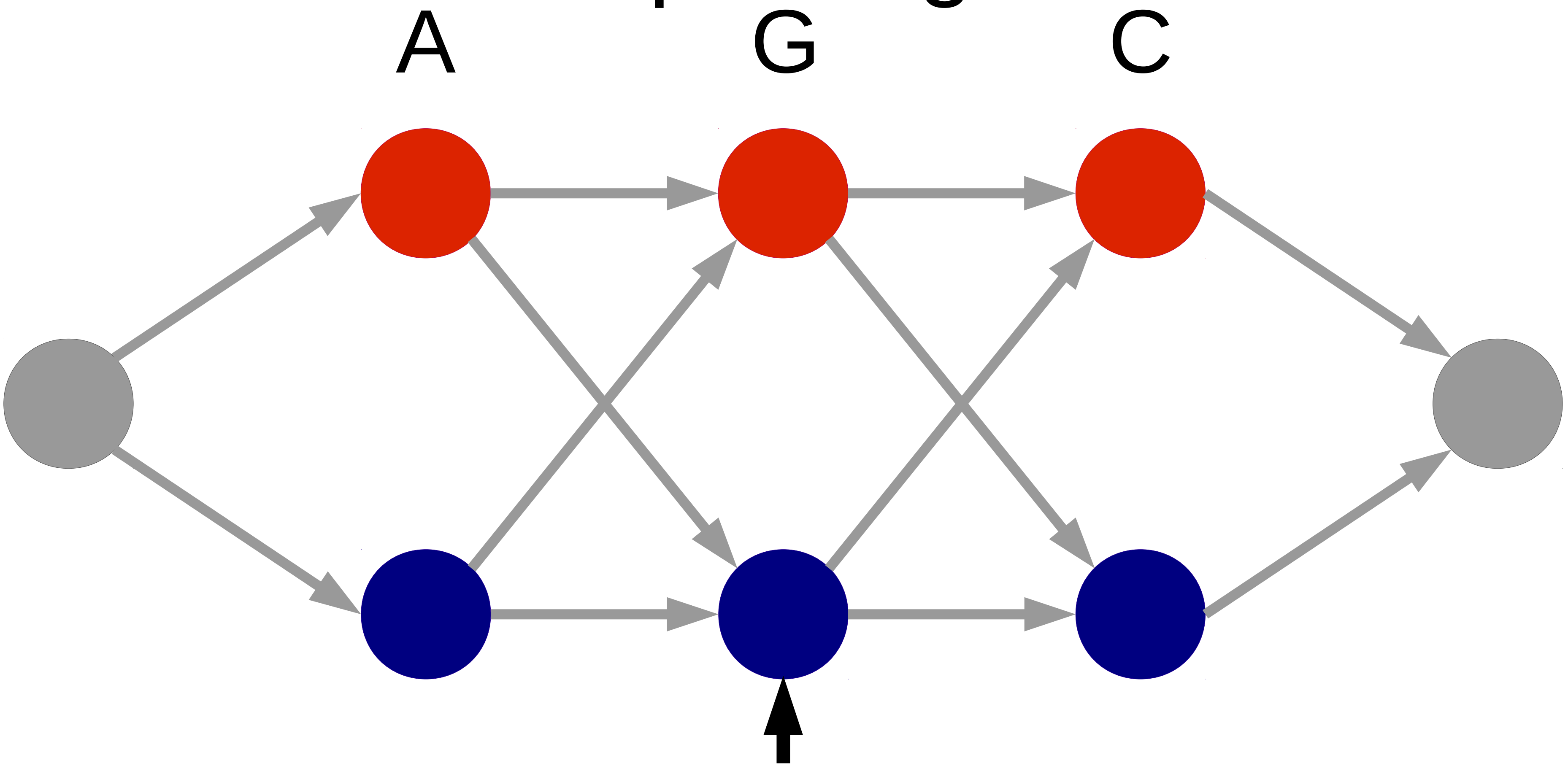

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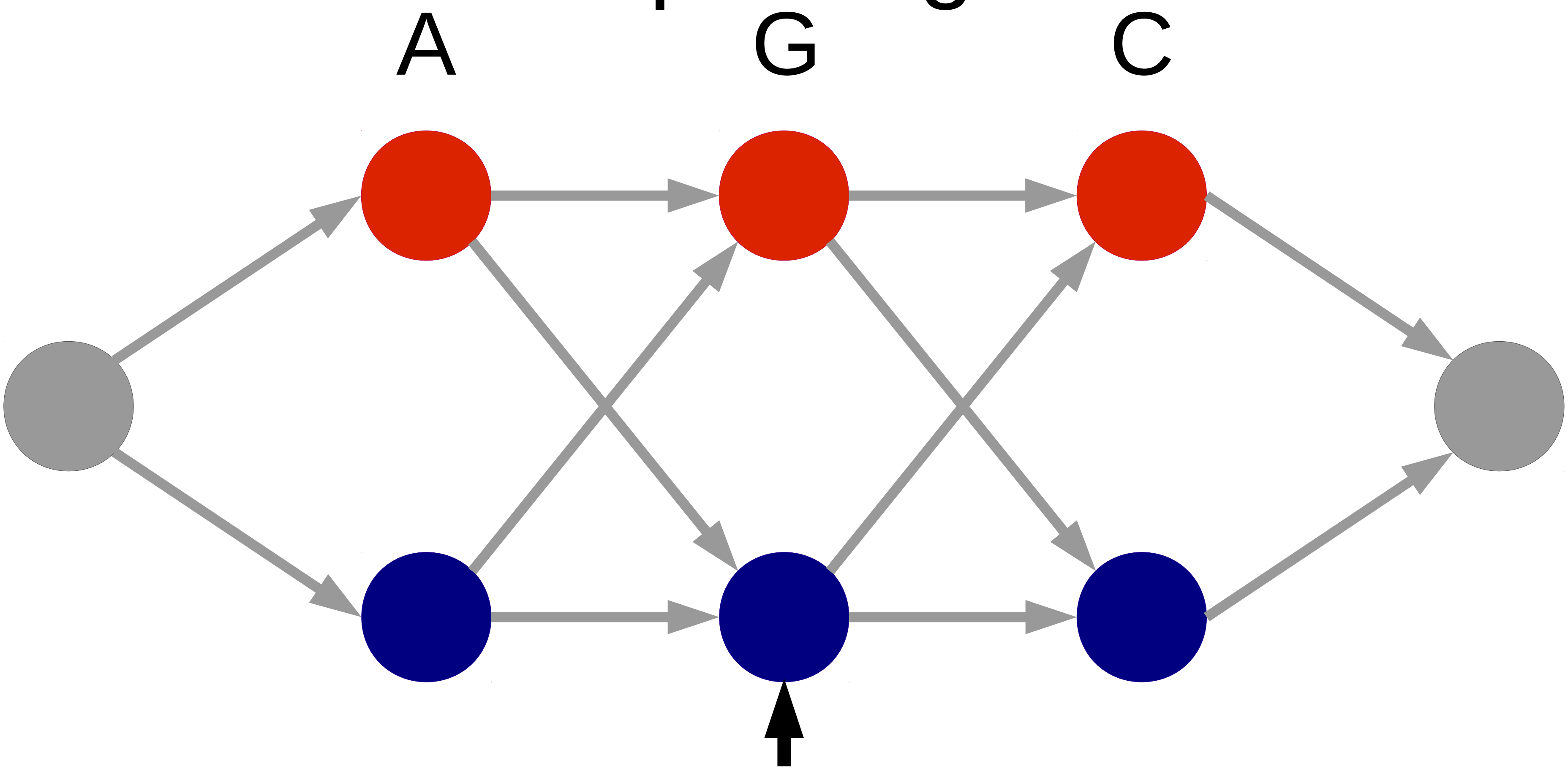

Consider the probabilities at each position:

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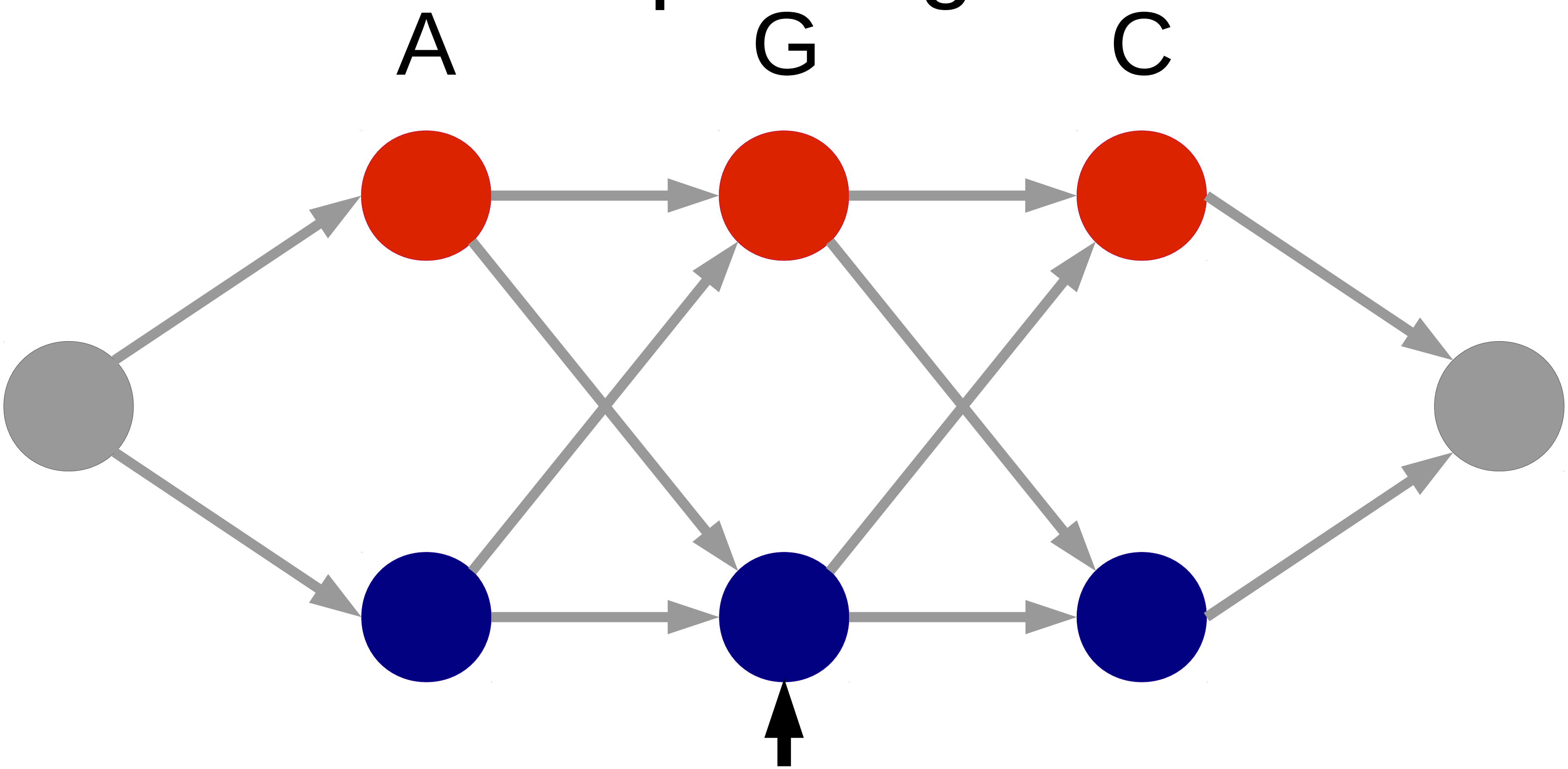

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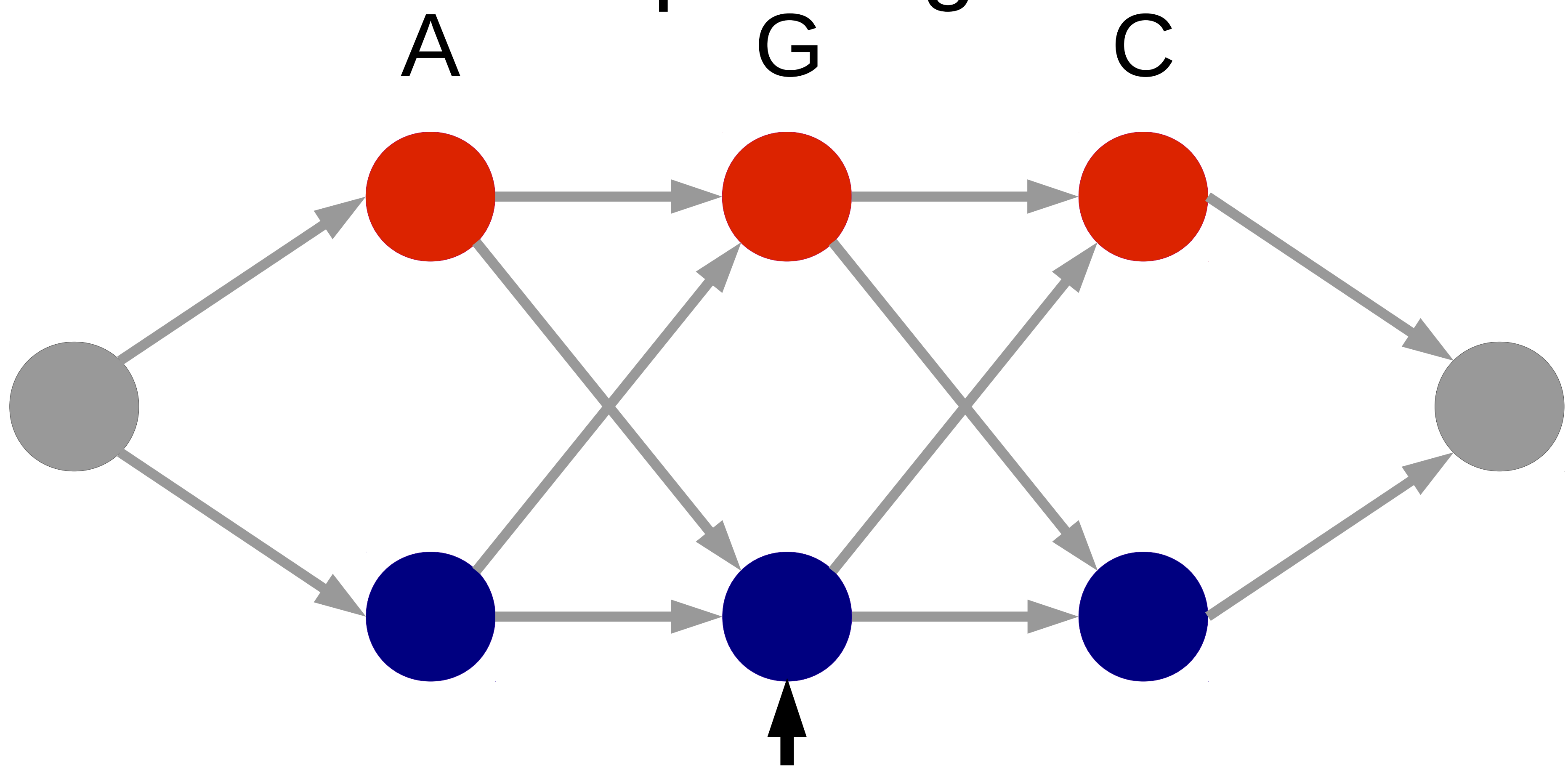

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$$\gamma_k(i) = \frac{\alpha_k(i) e_k(S_i) \beta_k(i)}{\sum_{j=1}^{N} \alpha_j(i) e_j(S_i) \beta_j(i)}$$

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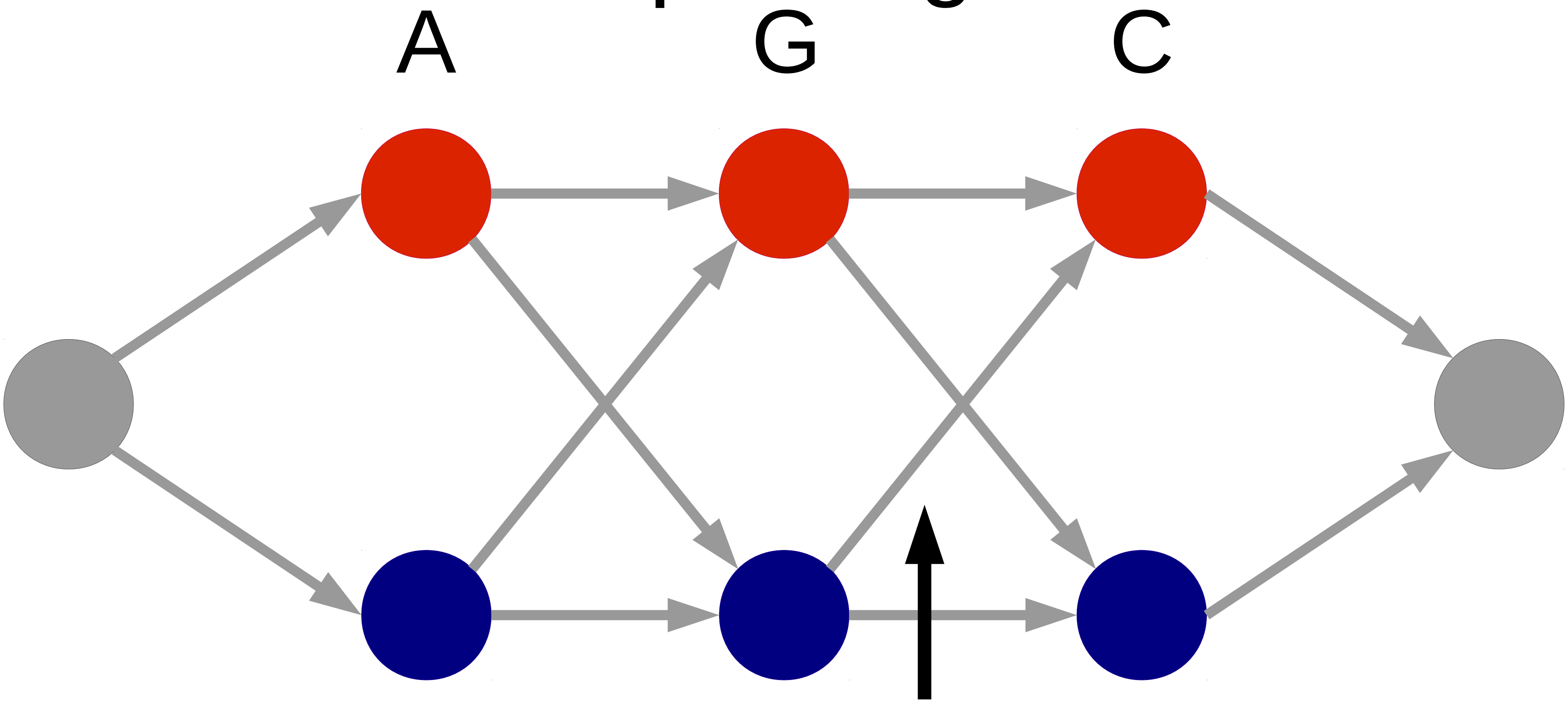

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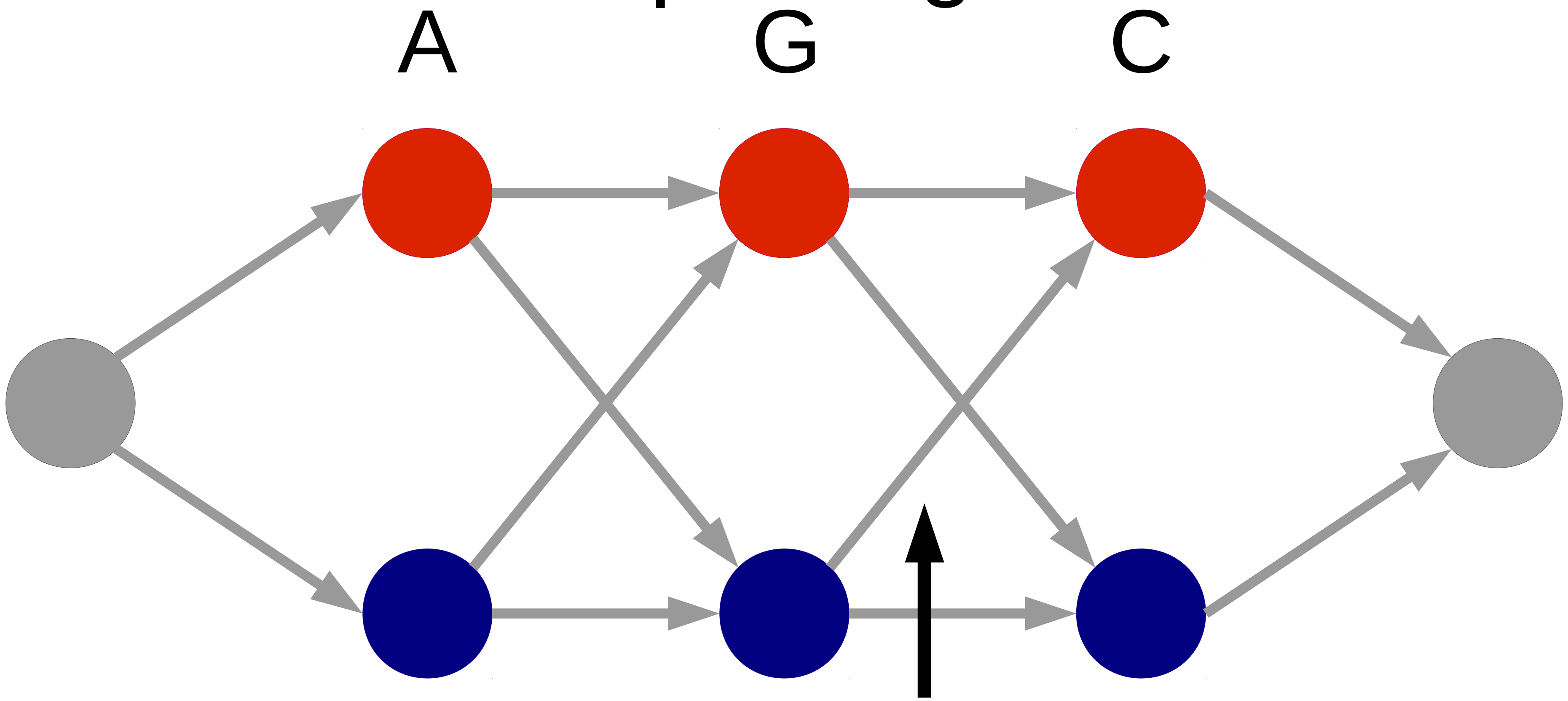

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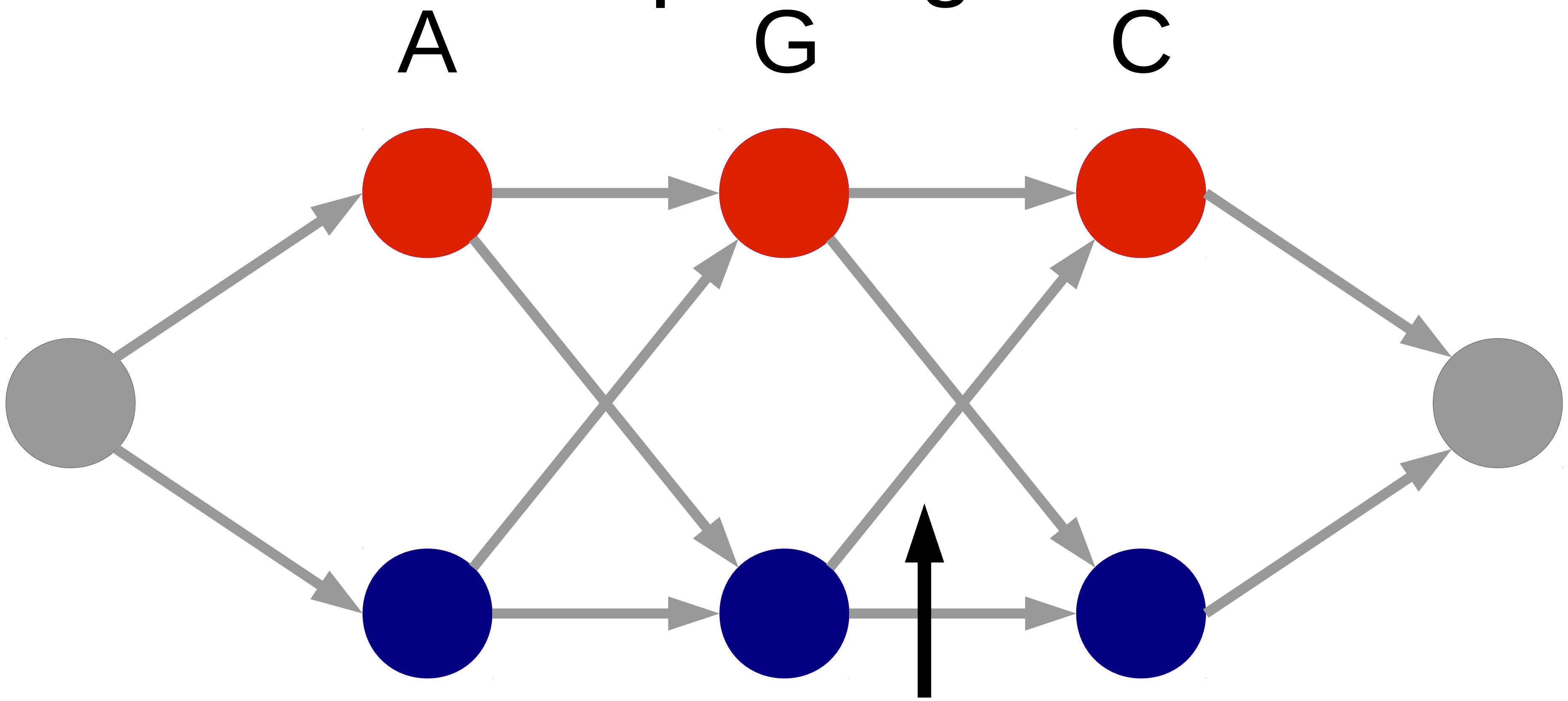

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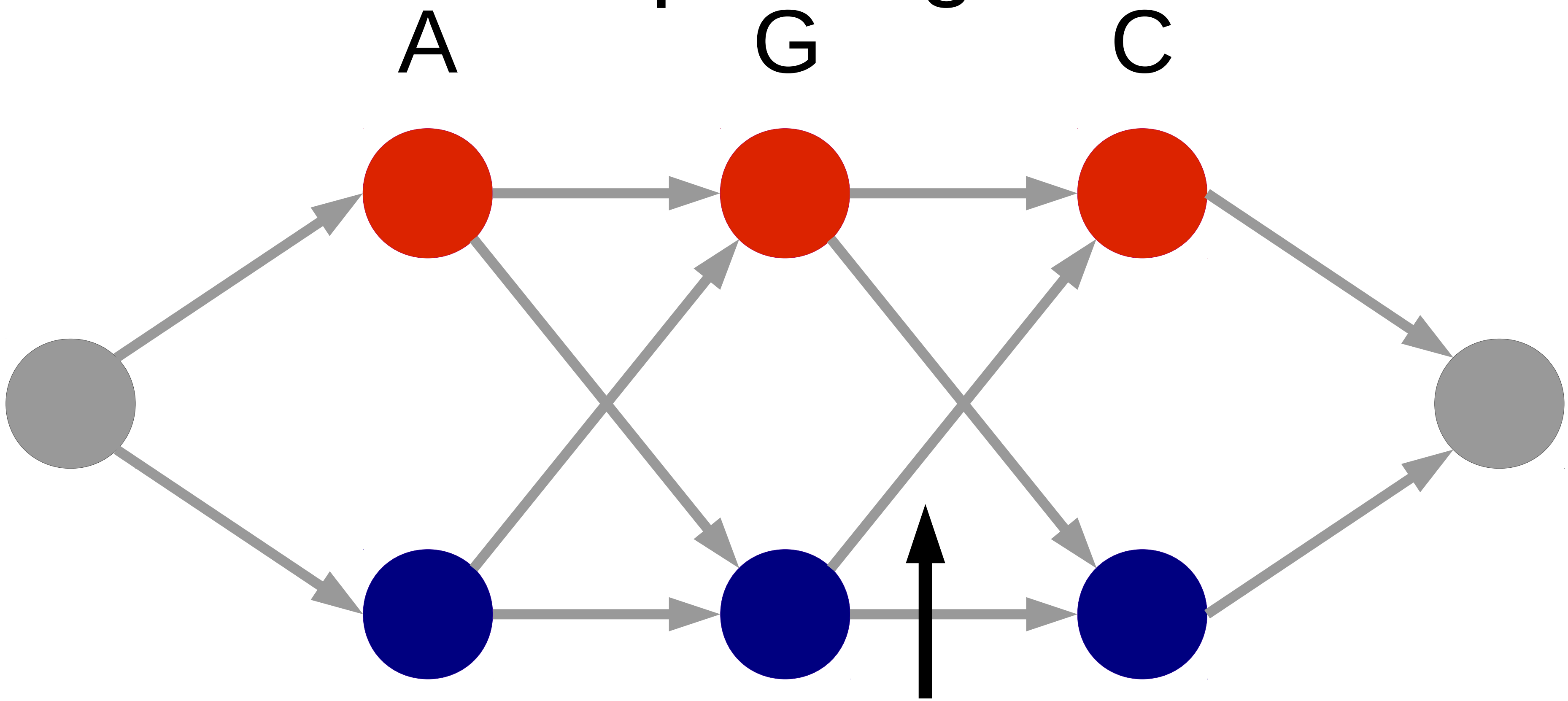

Consider the probabilities at each position:

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$$\xi_{kl}(i) = \frac{\alpha_k(i) e_k(S_i) a_{kl} e_l(S_{i+1}) \beta_l(i+1)}{\sum_{m=1}^{N} \sum_{n=1}^{N} \alpha_m(i) e_m(S_i) a_{mn} e_n(S_{i+1}) \beta_n(i+1)}$$

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$$1_{S_i=v}=\begin{cases}1 & \text{if } S_i=v\\ 0 & \text{otherwise}\end{cases}$$

Notes for debugging

1) Try calculating some simple forward and backward probabilities by hand to check your algorithm

2) Make sure the sum of the numerators for a single state or transition from a given state equals the associated denominator

3) The likelihood at each iteration should increase, if it decreases then you have a bug

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- Open question: Does P = NP?

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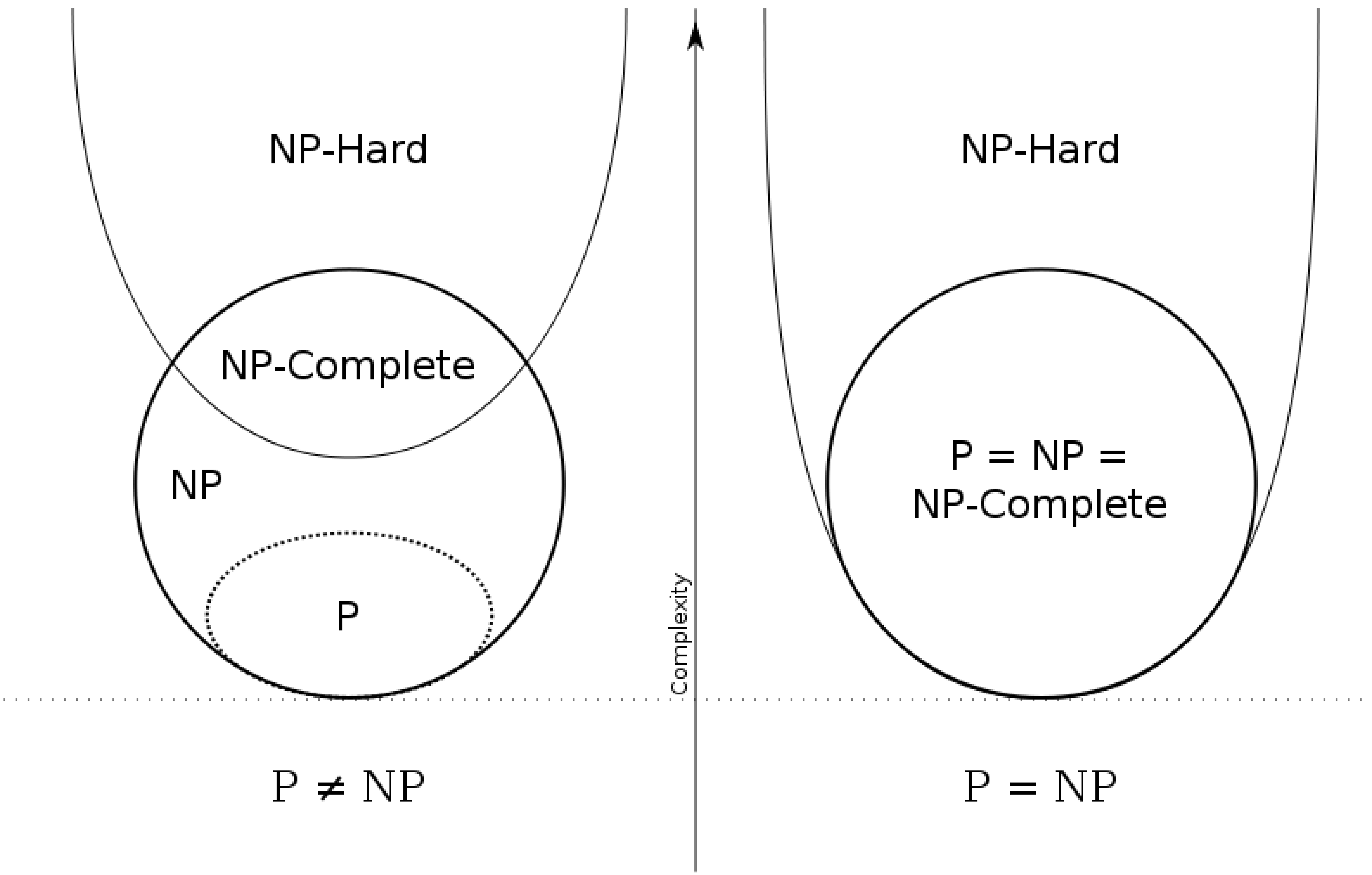

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- This doesn't necessarily mean you should give up, approximate P algorithms may exist for your NP problem

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4) Prove your reduction algorithm is in P

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