Discussion Section 8

- Baum-Welch

- NP-completeness proofs (or how to say “actually, this probably can't be done efficiently”)
Viterbi and Baum-Welch are maximizing different functions
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$$\max_{p} P_{\theta}(p, S)$$
Viterbi and Baum-Welch are maximizing different functions

- Viterbi likelihood:

  \[
  \max_P P_\theta(p, S)
  \]

- Baum-Welch likelihood:

  \[
  \sum_P P_\theta(p, S)
  \]
Baum-Welch
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1) Use **forward algorithm** to find log likelihood of the sequence (i.e. sum of all paths)
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2) Use **forward-backward** to get fractional counts for each edge type
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2) Use **forward-backward** to get fractional counts for each edge type

   \[
   \frac{(\text{total probability of paths passing through edge})}{(\text{total probability of all paths})}
   \]
Baum-Welch

1) Use **forward algorithm** to find log likelihood of the sequence (i.e. sum of all paths)

2) Use **forward-backward** to get fractional counts for each edge type
   
   \[ \frac{(\text{total probability of paths passing through edge})}{(\text{total probability of all paths})} \]

3) Re-estimate transition and emission probabilities by calculating the expected number of each edge type
Forward-backward algorithm
Forward-backward algorithm
For each node:
- Forward: Store the sum of probabilities of paths ending at position $i$ state $k$
For each node:
- Forward: Store the sum of probabilities of paths ending at position \( i \) state \( k \)
- Backward: Store the sum of probabilities of paths starting at position \( i \) state \( k \)
Forward-backward algorithm
Forward-backward algorithm

Total probability of paths passing through position $i$ state $k$: 
Forward-backward algorithm

Total probability of paths passing through position $i$ state $k$:
- $\text{forward}(i, k) \times \text{emission}(S_i, k) \times \text{backward}(i, k)$
Total probability of paths passing through position $i$ state $k$:
- $\text{forward}(i, k) \times \text{emission}(S_i, k) \times \text{backward}(i, k)$
- In this example, add this weighted count to the numerator for the blue state emitting 'G' and the denominator for all blue state emission probabilities
Forward-backward algorithm
Forward-backward algorithm

A

G

C

Total probability of paths passing from position $i-1$ state $k'$ to position $i$ state $k$: 
Total probability of paths passing from position $i-1$ state $k'$ to position $i$ state $k$:
- $\text{forward}(i-1, k') \times \text{emission}(S_{i-1}, k') \times \text{transition}(k', k) \times \text{emission}(S_i, k) \times \text{backward}(i, k)$
Forward-backward algorithm

Total probability of paths passing from position $i$-1 state $k'$ to position $i$ state $k$:

- $\text{forward}(i-1, k') \times \text{emission}(S_{i-1}, k') \times \text{transition}(k', k) \times \text{emission}(S_i, k) \times \text{backward}(i, k)$
- In this example, add this weighted count to the numerator for the transitions from blue to red and the denominator for all transition out of blue states
An alternative way to think about updating
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- Some terminology for the following slides
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  - $\alpha_k(i)$: The forward probability of being in state $k$ at position $i$
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- Some terminology for the following slides
  - $\alpha_k(i)$: The forward probability of being in state $k$ at position $i$
  - $\beta_k(i)$: The backward probability of being in state $k$ at position $i$
An alternative way to think about updating

- Some terminology for the following slides
  - $\alpha_k(i)$: The forward probability of being in state $k$ at position $i$
  - $\beta_k(i)$: The backward probability of being in state $k$ at position $i$
  - $e_k(S_i)$: The emission probability of the character at position $i$ in state $k$
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• Some terminology for the following slides
  - $\alpha_k(i)$: The forward probability of being in state $k$ at position $i$
  - $\beta_k(i)$: The backward probability of being in state $k$ at position $i$
  - $e_k(S_i)$: The emission probability of the character at position $i$ in state $k$
  - $a_{kl}$: The transition probability from state $k$ to state $l$
An alternative way to think about updating
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Consider the probabilities at each position:
An alternative way to think about updating

Consider the probabilities at each position:
- figure out the probability of being in state $k$ at position $i$
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$$
\gamma_k(i) = \frac{\alpha_k(i) e_k(S_i) \beta_k(i)}{\sum_{j=1}^{N} \alpha_j(i) e_j(S_i) \beta_j(i)}
$$
An alternative way to think about updating
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Consider the probabilities at each position:
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Consider the probabilities at each position:

- figure out the probability of going from state $k$ to state $l$ from position $i$ to position $i+1$
An alternative way to think about updating

Consider the probabilities at each position:
- figure out the probability of going from state $k$ to state $l$ from position $i$ to position $i+1$

$$
\xi_{kl}(i) = \frac{\alpha_k(i) e_k(S_i) a_{kl} e_l(S_{i+1}) \beta_l(i+1)}{\sum_{m=1}^{N} \sum_{n=1}^{N} \alpha_m(i) e_m(S_i) a_{mn} e_n(S_{i+1}) \beta_n(i+1)}
$$
An alternative way to think about updating
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- The initial probabilities for each state $k$ can be updated to $\gamma_k(1)$
An alternative way to think about updating

• The initial probabilities for each state $k$ can be updated to

$$\gamma_k(1)$$

• The transition probability from state $k$ to state $l$ can be updated to

$$\frac{\sum_{i=1}^{\xi_{kl}(i)}}{\sum_{i=1}^{\gamma_k(i)}}$$
An alternative way to think about updating

- The initial probabilities for each state $k$ can be updated to
  \[ \gamma_k(1) \]

- The transition probability from state $k$ to state $l$ can be updated to
  \[ \sum_{i=1}^{\xi_{kl}(i)} \frac{\xi_{kl}(i)}{\sum_{i=1}^{\gamma_k(i)}} \]

Remember to ignore the last position.
An alternative way to think about updating

• The initial probabilities for each state $k$ can be updated to

$$
\gamma_k(1)
$$

• The transition probability from state $k$ to state $l$ can be updated to

$$
\sum_{i=1}^{\sum_{i=1}} \xi_{kl}(i) \frac{\gamma_k(i)}{\sum_{i=1}^{\sum_{i=1}} \gamma_k(i)}
$$

Remember to ignore the last position

• The emission probability for symbol $v$ from state $k$ can be updated to

$$
\sum_{i=1}^{\sum_{i=1}} 1_{S_i=v} \gamma_k(i) \frac{\gamma_k(i)}{\sum_{i=1}^{\sum_{i=1}} \gamma_k(i)}
$$
An alternative way to think about updating

- The initial probabilities for each state $k$ can be updated to
  \[ \gamma_k(1) \]
- The transition probability from state $k$ to state $l$ can be updated to
  \[
  \frac{\sum_{i=1}^{\xi_{kl}}(i)}{\sum_{i=1}^{\gamma_k(i)}}
  \]
  Remember to ignore the last position
- The emission probability for symbol $v$ from state $k$ can be updated to
  \[
  \frac{\sum_{i=1}^{1_{S_i=v}\gamma_k(i)}}{\sum_{i=1}^{\gamma_k(i)}}
  \]
  \[1_{S_i=v} = \begin{cases} 
  1 & \text{if } S_i = v \\
  0 & \text{otherwise}
  \end{cases}\]
Notes for debugging

1) Try calculating some simple forward and backward probabilities by hand to check your algorithm.

2) Make sure the sum of the numerators for a single state or transition from a given state equals the associated denominator.

3) The likelihood at each iteration should increase; if it decreases then you have a bug.
Formal definition of P, NP, and NP-hard
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- **Open question**: Does $P = NP$?
Formal definition of $P$, $NP$, and $NP$-hard

$P \neq NP$

$P = NP = NP$-Complete

Complexity
How to prove that a problem is NP-complete
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• This doesn't necessarily mean you should give up, approximate P algorithms may exist for your NP problem
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  1) Prove the problem is in NP
  2) Construct an algorithm to transform a known NP-complete problem into your problem
  3) Prove that solutions to your problem are correct if and only if they are solutions to the reduced NP-complete problem
  4) Prove your reduction algorithm is in P
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- There are many problems that have been proven to be NP-complete that you can select from
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  - Classic Nintendo games (again, check out arXiv)