## Lecture 9: <br> Sequence Alignment

- Sequence alignment and evolution
- mutations
- Edit graph \& alignment algorithms
- Multiple sequence alignment
- Higher-dimensional edit graphs
- Progressive alignment


## Aligning sequences: Major uses in genome analysis

- To find relationship between sequences from "same" genome, e.g.
- finding gene structure by aligning cDNA to genome
- assembling sequence reads in genome sequencing project
- NextGen applications: "Resequencing", ChIPSeq, etc Still need to allow for discrepancies
- due to basecalling errors \& polymorphisms, introns but exact match methods (hashtables, suffix arrays) do most of the work
- To detect evolutionary relationships among sequences:
- illuminating protein structure and function via distant matches
- illuminating mutation and selection in genomes
- helps find non-neutrally evolving (functional) regions

Here, frequent discrepancies make finding the alignment more challenging

- Often we're interested in details of alignment
- (i.e. precisely which residues are aligned),
but
- sometimes only care whether alignment score is large enough to imply sequences are related


## Sequences \& evolution

- Similar sequences of sufficient length usually have a common evolutionary origin
- i.e. are homologous
- For a pair of sequences
- "\% similarity" makes sense
- "\% homology" doesn't
- In alignment of two homologous sequences
- differences mostly represent mutations that occurred in one or both lineages, but
- Not all mutations are inferrable from the alignment


## Mutation types

- single-base substitution error by DNA polymerase - most common type?
- strand slippage error by polymerase, inserting or deleting one or more bases
- DNA damage (radiation, or chemical) + errorprone repair, possibly altering more than one nucleotide, e.g.
- CpG (hydrolytic deamination of methyl C)
- dinucleotide changes, perhaps UV-induced dipyrimidine lesions (Science 287: 1283-1286)
- Rearrangements (break and rejoin)
- Inversion (2 breaks on same chromosome)
- Translocation (2 breaks on different chromosomes)
- More complex (> 2 breaks)
- Duplication of a segment
- Deletion of a segment
- Insertion/excision of transposable element
- Acquisition of DNA from another organism ("horizontal transfer")


## Mutation rates may depend on:

- lineage (organism): no universal "molecular clock"
- sex: e.g. in mammals, mut rate higher in males than females
- type of change - e.g.
- replacement ("substitution") of one nucleotide by another more freq than indels (insertions or deletions)
- transition replacements
- pyrimidine $\rightarrow$ pyrimidine $(\mathrm{T} \leftrightarrow \mathrm{C}$ ), or purine $\rightarrow$ purine $(\mathrm{A} \leftrightarrow \mathrm{G})$ more freq than transversion replacements
- pyrimidine $\rightarrow$ purine, or purine $\rightarrow$ pyrimidine
- GC or AT bias in some organisms
- e.g. $\mathrm{G} \rightarrow \mathrm{A}$ more freq than $\mathrm{A} \rightarrow \mathrm{G}$ in most eukaryotes
- causes most genomes to be relatively $\mathrm{A}+\mathrm{T}$ rich
- (small) deletions generally more frequent than (small) insertions
- sequence context (e.g. CpG effect)
- position in sequence - some sites more slowly changing than others, due to
- selection - e.g. in coding sequences,
- indels strongly selected against because would disrupt reading frame;
- non-synonymous changes less freq than synonymous
- variation in underlying mutation rate (cf. mouse genome paper)
- may in part depend on replication timing (late replication less accurate)
- typical per base subst rates in non-coding DNA:
$-\sim 1 \times 10^{-9}$ per base per year (order of magnitude)
- in humans, about $10^{-9}$ / base / year, $\Rightarrow 2 \times 10^{-8} /$ base / generation
$\Rightarrow 120$ / diploid genome / generation
(recent de novo estimates are lower!)
- freq of gene duplication is $\sim 10^{-8}$ per gene per year (Science 290: 1151-1155)
- freq of simultaneous dinuc substitutions is $\sim 10^{-10}$ per dinuc site per year (Science 287: 1283-1286)
- freq of $\mathrm{CpG} \Rightarrow \mathrm{TpG}$ or CpA changes is $\sim 10$-fold higher (per CpG ) than other substs in mammalian DNA;
- may account for $\sim 20 \%$ of all substitutions.
(Observed) ALIGNMENT:
... acagaatcagggtcccgtta...
(may not be unique!)
...accgaatcagg-tcccgtca...
(Unobserved) MUTATION HISTORY (in general, this is not even inferrable!): ...accgaatcgggtcccgtta...
...acagaatcgggtcccgtta...
...acagaatcaggtcccgtta...

...accgaatcaggtcccgtta...

...acagaatcagggtcccgtta...


## Complications

- Parallel \& back mutations
$\Rightarrow$ estimating total \# of mutations requires statistical modelling
- Segmental mutations
- duplications \& other large indels
- inversions
are not well modelled by alignments
- genome-scale alignments usually done 'in pieces'


## Sequence alignments correspond to paths in a $D A G$ !

The Edit Graph for a Pair of Sequences


- The edit graph is a DAG.
- Except on the boundaries, the nodes have in-degree and out-degree both 3 .
- The depth structure is as shown on the next slide. Child of node of depth $n$ always has
- depth $n+1$ (for a horizontal or vertical edge), or
- depth $n+2$ (for a diagonal edge).

- Paths in edit graph correspond to alignments of subsequences
- each edge on path corresponds to an alignment column
- diagonal edges correspond to column of two aligned residues
- horizontal edges correspond to column with
- residue in $1^{\text {st }}$ (top, horizontal) sequence
- gap in the $2^{\text {d }}$ (vertical) sequence
- vertical edges correspond to column with
- residue in $2^{\mathrm{d}}$ sequence
- gap in $1^{\text {st }}$ sequence


Above path corresponds to following alignment (w/ lower case letters considered unaligned):

$$
\begin{aligned}
& \text { aCGTTGAATGAccca } \\
& \text { gCAT-GAC-GA }
\end{aligned}
$$

## Weights on Edit Graphs

- Edge weights correspond to scores on alignment columns.
- Highest weight path corresponds to highest-scoring alignment for that scoring system.
- Weights may be assigned using
- a substitution score matrix
- assigns a score to each possible pair of residues occurring as alignment column
- or profile
- scores specific to a particular sequence
and
- a gap penalty
- assigns a score to column consisting of residue opposite a gap.
based on appropriate probability models (next lecture!)


## Alignment algorithms

- Smith-Waterman algorithm to find highest scoring alignment
$=$ dynamic programming algorithm to find highestweight path
- is a local alignment algorithm:
- finds alignment of subsequences rather than the full sequences.
- Can process nodes in any order in which parents precede children. Commonly used alternatives are
- depth order
- row order
- column order



## Complexity

- For two sequences of lengths $M$ and $N$, edit graph has
- $(M+1)(N+1)$ nodes,
- $3 M N+M+N$ edges,
- time complexity: $O(M N)$
- space complexity to find
highest score and beginning \& end of alignment is $O(\min (M, N))$
(since only need store node's values until children processed)
- space complexity to reconstruct highest-scoring alignment: $O(M N)$
- For genomic comparisons may have
$-M, N \approx 10^{6}$ (if comparing two large genomic segments), or
$-M \approx 10^{3}, N \approx 10^{9}$ (if searching gene sequence against entire genome);
in either case $M N \approx 10^{12}$.
- Time complexity $10^{12}$ is (marginally) acceptable.
- $\exists$ speedups which reduce constant by
- reducing calculations per matrix cell, using fact that score often 0
- (our program swat).
- still guaranteed to find highest-scoring alignment.
- reducing \# cells considered, using nucleating word matches
- (BLAST, or cross_match).
- Lose guarantee to find highest-scoring alignment.


## Multiple sequence alignment

- More sequences =>
- (potentially) more accurate alignments
- better resolution of mutations, selection
- Need $>2$ sequences to polarize mutations
- An evolutionary tree relates the sequences!


The Edit Graph for a Pair of Sequences


## Multiple Alignment via Dynamic Programming

- Higher dimension edit graph
- each dimension corresponds to a sequence; co-ordinates labelled by residues
- Each edge corresponds to aligned column of residues (with gaps).
- Can put arbitrary weights on edges; in particular,
- can make these correspond to probabilities under an evolutionary model (Sankoff 1975).
- implicitly assumes independence of columns
- Highest weight path through graph again gives optimal alignment


## Generalization to Higher Dimension

Each "cell" in 3-dimensional case looks like this:


Each edge projects onto a gap or residue in each dimension, defining an alignment column; e.g. red edge defines

```
V
M
```

- \# edges \& \# vertices are proportional to product of sequence lengths.
- For $k$ sequences of size $N$, is of order $O\left(N^{k}\right)$
- impractical even for proteins ( $N \sim 300$ to 500 residues) if $k>5$ :

$$
300^{5}=2.4 \times 10^{12}
$$

## Multiple alignments: paths in huge WDAGs

- To find high-scoring paths, need to
- reduce size of graph
- restrict allowed weighting schemes, and/or
- sacrifice optimality guarantees
- Durbin et al. discuss methods implementing these ideas:
- Hein
- Carillo-Lipman
- progressive alignment (e.g. Clustal)
- HMMs provide nice (but not guaranteed optimal) approach for constructing multiple alignments


## Progressive alignment

- Simplest version: align one sequence (the reference) to each of the others, pairwise; construct multiple alignment from that.
- More generally, progressively align pairs of (sequences or) alignments, using a guide tree - Tree may reflect evolution, or sequence quality
- Will tend to be more accurate
- Revise gaps
- correct errors due to gap placement \& gap attraction


## Guide Tree



- Complexity: $\mathrm{N}^{2} \times(n-1)$ where
$-\mathrm{N}=$ seq length, $n=\#$ seqs
instead of $\mathrm{N}^{n}$
- (does not count gap correction)

